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Steps towards full two-loop calculations for 2 fermion to 2 fermion processes: running versus pole masses schemes

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Recent progress in the calculation of the two-loop on-shell mass counterterms within the electroweak Standard Model (SM) for the massive particles are discussed. We are in progress of developing a package for full two-loop SM calculations of $2 \to 2$ fermion processes, with emphasis on the analytical approach where feasible. The complete two-loop on-shell renormalization is implemented. Substantial progress has been made in calculating the master integrals. We are able to compute in an efficient and stable manner up to a few thousands of diagrams of very complex mass structure.

1. Introduction

Even for the simplest physical processes like $e^+e^- \to f\bar{f}$ complete two-loop electroweak SM calculations still are not available, maninly because of the enourmous complexity of such calculations. An exception is the μ–decay rate [1], which is simpler due to the fact that it is a static quantity. Besides the large number of diagrams encountered, the difficulties start at the level of individual Feymnan integrals and increase substantially when going from propagators to vertices or box contributions. One key problem at the beginning is the renormalization and the calculation of the necessary counterterms. In the QED like on-shell renormalization scheme the basic input parameters for electroweak higher order calculations are the fine structure constant $\alpha = \frac{e^2}{4\pi}$ and all the physical particle masses. In this scheme the whole renormalization program only requires the calculation of selfenergy diagrams and tadpoles. While the calculation of the counterterm for the fine structure constant is relatively easy (at zero momentum) [2], the on-shell mass counterterms are much more involved. In this note we therefore focus on aspects of calculating the latter, which yields the relation between bare, $\overline{\rm MS}$ and on-shell (pole) masses (two-loop renormalization constant in on-shell scheme). Many details on this program may be found in the original publications [3,4] and will not be repeated here. We rather concentrate on some controversal points concerning the definition of $\overline{\rm MS}$ masses in electroweak theory (Sec. 2). We also give some more specific information concerning our numerical approach to the calculation of huge sets of diagrams (Sec. 3).

2. $\overline{\rm MS}$-masses of particles in SM and renormalization group equations

The two-loop calculation of pole masses of the gauge bosons in the SM has been discussed in [3,4]. In terms of the transversal self-energy function $\Pi(p^2, m^2, \cdots)$, by expansion about $p^2 = -m^2$, will get the two loop solution

$$s_P = m^2 - \Pi^{(1)} - \Pi^{(2)} - \Pi^{(1)}\Pi^{(1)\prime} \,, \quad (1)$$

for the location of the pole s_P. $\Pi^{(L)}$ is the bare or $\overline{\rm MS}$-renormalized L-loop contribution to Π, the prime denotes the derivative with respect to p^2. One of the remarkable properties of (1) is that the complex pole is represented by the self-energy and its derivative at momentum equal to the bare or $\overline{\rm MS}$ mass which, by definition, are real parameters. The standard parametrization of the pole is $s_{P,a} = M_a^2 - iM_a\Gamma_a$, where M_a is the pole mass and Γ_a is the width of particle a.

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the SM two renormalization schemes are commonly accepted: the on-shell and the $\overline{\rm MS}$ scheme. It is well understood, that in the on-shell scheme all momentum independent diagrams, in particular the tadpoles, can be omitted (the on-shell scheme is a particular case of a momentum subtraction scheme: finite parts are fixed by subtraction of the propagators at $p^2 = -s_P$). The set of diagrams contributing to the $\overline{\rm MS}$ mass are not unambiguously defined. Let us first express the pole (1) in terms of the bare amplitude in a manifestly gauge invariant manner. This requires to include the Higgs tadpole contribution [5]. Only this complete gauge invariant bare amplitude should be utilized as a starting point to set up $\overline{\rm MS}$ renormalization. At the two-loop level $\overline{\rm MS}$ renormalization can be written as

$$
\begin{aligned}
s_P &= m_0^2 - \Pi_0^{(1)} - \Pi_0^{(2)} - \Pi_0^{(1)} \Pi_0^{(1)\prime} \\
&- \left[\sum_j (\delta m_{j,0}^2)^{(1)} \frac{\partial}{\partial m_{j,0}^2} + \sum_j (\delta g_{j,0})^{(1)} \frac{\partial}{\partial g_{j,0}} \right] \Pi_0^{(1)} \\
&= m_a^2 - \left\{ \Pi_a^{(1)} \right\}_{\overline{\rm MS}} - \left\{ \Pi_a^{(2)} + \Pi_a^{(1)} \Pi_a^{(1)\prime} \right\}_{\overline{\rm MS}}
\end{aligned}
\tag{2}
$$

where the sum runs over all species of particles, $g_j = \alpha,\ g_s,\ (\delta g_{j,0})^{(1)}$ and $(\delta m_{j,0}^2)^{(1)}$ are the one-loop counterterms for the charges and physical masses in the $\overline{\rm MS}$ -scheme and after differentiation we put all parameters equal to their on-shell values. The derivatives in Eq. (2) correspond to the subtraction of sub-divergencies. The genuine two-loop mass counterterm comes from the shift of the m_0^2 term. The relation between bare- and $\overline{\rm MS}$ -masses has the form

$$
m_{a,0}^2 = m_a^2(\mu)\, (1 + \sum_{k=1} Z_a^{(k)} \varepsilon^{-k}) \, . \tag{3}
$$

To renormalize the pole mass at the two-loop level requires to calculate the one-loop renormalization constants for all physical parameters (charge and masses), and the two-loop renormalization constant only for the mass itself. Not needed are the wave-function renormalization or ghost (unphysical) sector renormalizations. After UV-renormalization the pole is represented in terms of finite amplitudes. Now, expression (2) connects the pole s_P with the $\overline{\rm MS}$ parameters: masses and charges. This expression can be inverted and solved iteratively. The solution to two-loop reads

$$
\begin{aligned}
m_a^2 &= M_a^2 + {\rm Re} \left\{ \Pi_a^{(1)} \right\}_{\overline{\rm MS}} + {\rm Re} \left\{ \Pi_a^{(2)} + \Pi_a^{(1)} \Pi_a^{(1)\prime} \right\}_{\overline{\rm MS}} \\
&+ \left[(\Delta e)^{(1)} \frac{\partial}{\partial e} + \sum_j (\Delta m_j^2)^{(1)} \frac{\partial}{\partial m_j^2} \right] {\rm Re} \left\{ \Pi_a^{(1)} \right\}_{\overline{\rm MS}}
\end{aligned}
\tag{4}
$$

where the sum runs over all species of particles $j = Z, W, H, t,\ (\Delta m_j^2)^{(1)} = {\rm Re} \left\{ \Pi_j \right\}_{\overline{\rm MS}}$, and the transition from the $\overline{\rm MS}$ to the on-shell scheme for the electric charge [6] is also included. The mass on the l.h.s. of this expression we call the $\overline{\rm MS}$-mass of particle. It should be noted, that in this definition the tadpole contribution does not cancel, so that higher powers of the Higgs and the top-quark mass show up at higher orders. In particular, at two-loops, the purely bosonic diagrams generate m_H^4/m_V^4 terms and the third fermion family gives rise to the appearance of $m_t^6/(m_H^2 m_V^4)$ power corrections. For the $\overline{\rm MS}$-masses, defined in this way, the following properties are valid:

1. The UV counter-terms satisfy relations connecting the higher order poles with the lower order ones:

$$
\begin{aligned}
&\left(\gamma_a + \sum_j \beta_{g_j} \frac{\partial}{\partial g_j} + \sum_i \gamma_i m_i^2 \frac{\partial}{\partial m_i^2} \right) Z_a^{(n)} \\
&= \frac{1}{2} \sum_j g_j \frac{\partial}{\partial g_j} Z_a^{(n+1)} \, ,
\end{aligned}
\tag{5}
$$

where we adopt the following definitions for the RG functions: for all dimensionless coupling constants, like $g, g', g_s, e, \lambda, y_t$, the β-function is given by $\mu^2 \frac{\partial}{\partial \mu^2} g = \beta_g$ and for all mass parameters (a mass or the Higgs v.e.v. v) the anomalous dimension γ_{m^2} is given by $\mu^2 \frac{\partial}{\partial \mu^2} \ln m^2 = \gamma_{m^2}$.

2. Using the fact that s_P is RG-invariant: $\mu^2 \frac{d}{d\mu^2} s_P \equiv 0$, we are able to calculate the anomalous dimension of the masses from our finite results (4) or from the UV counterterms (3)

$$
\gamma_a = \sum_j \frac{1}{2} g_j \frac{\partial}{\partial g_j} Z_a^{(1)} \, , (j = g, g_s).
$$

3. All tree level relations between masses of any particles and parameters of the unbroken Lagrangian are RG invariant. This means, in par-

ticular, that the RG equation for the vacuum expectation value v is given by $\gamma_{v^2} \equiv \gamma_{m^2} - \beta_\lambda/\lambda$, where m^2 and λ are the parameters of the symmetric scalar potential. This fact allow to get anomalous dimension of the masses via the relations [7]

$$\gamma_W = \gamma_{m^2} - \frac{\beta_\lambda}{\lambda} + 2\frac{\beta_g}{g} \ ,$$

$$\gamma_Z = \gamma_{m^2} - \frac{\beta_\lambda}{\lambda} + 2\left(c_W \frac{\beta_g}{g} + s_W \frac{\beta_{g'}}{g'} \right) ,$$

$$\gamma_t = \gamma_{m^2} - \frac{\beta_\lambda}{\lambda} + \frac{\beta_{y_t}}{y_t} \ , \quad \gamma_H = \gamma_{m^2} \ , \tag{6}$$

where $s_W(c_W)$ are the sin (cos) of the weak mixing angle and the 2-loop RG functions $\beta_g, \beta_{g'}, \beta_\lambda, \gamma_{m^2}, \beta_{y_t}$ are calculated in the unbroken phase [8].

The RG invariance of the pole positions s_P allow us to factorize explicitely the RG logarithms

$$M_a^2 = m_a^2 - \sum_j g_j^2 \left(m_a^2 \gamma_j^{(a)} L_b - X_j^{(a)} \right)$$
$$+ \sum_{i,j} g_i^2 g_j^2 \left[m_a^2 \left(C_{i,j;a}^{(2,2)} L_b^2 + C_{i,j;a}^{(2,1)} L_b \right) + X_{i,j}^{(a)} \right] ,$$

where $L_b = \ln \mu^2/m_b^2$, $C_{i,j;a}^{(m,n)} = C_{j,i;a}^{(m,n)}$ and

$$\mu^2 \frac{\partial}{\partial \mu^2} \ln m_k^2 = \gamma^{(k)} = \sum_j g_j^2 \gamma_j^{(k)} + \sum_{i,j} g_i^2 g_j^2 \gamma_{i,j}^{(k)}$$

$$\mu^2 \frac{\partial}{\partial \mu^2} g_k = \sum_j g_j^3 \beta_j + \sum_{i,j} g_i^2 g_j^2 \beta_{i,j} \ ,$$

$$C_{i,j;a}^{(2,1)} = \gamma_{i,j}^{(a)} + \frac{1}{2}\left(\gamma_i^{(a)} \gamma_j^{(b)} + \gamma_i^{(b)} \gamma_j^{(a)} \right) + 2\beta_j \delta_{i,j} \frac{X_j^{(a)}}{m_a^2}$$
$$+ \frac{1}{2} \sum_k \frac{m_k^2}{m_a^2} \left(\gamma_i^{(k)} \frac{\partial}{\partial m_k^2} X_j^{(a)} + \gamma_j^{(k)} \frac{\partial}{\partial m_k^2} X_i^{(a)} \right) \ ,$$

$$2C_{i,j;a}^{(2,2)} = 2\beta_j \gamma_j^{(a)} \delta_{i,j} + \gamma_i^{(a)} \gamma_j^{(a)}$$
$$+ \frac{1}{2} \sum_k \left(\gamma_i^{(k)} m_k^2 \frac{\partial}{\partial m_k^2} \gamma_j^{(a)} + \gamma_j^{(k)} m_k^2 \frac{\partial}{\partial m_k^2} \gamma_i^{(a)} \right) \ ,$$

where $X_j^{(a)}$ and their derivatives can be extracted from Appendixes of [5,3,4].

Crucial point of our definition of the $\overline{\rm MS}$ -mass (4) is the gauge invariant construction (1) for the pole in terms of the unrenormalized, bare diagrams. It can be done only after inclusion of the Higgs tadpole contribution. Another important ingredient are the Ward identities.

A. The inclusion of the tadpoles is necessary to ensure, that the physical Higgs field has zero vacuum expectation value in each order of the loop expansion.

B. It is well know, that in order to preserve the Ward identities for the longitudinal part of the gauge boson propagator it is necessary to add the tadpole contribution, which is equal to the propagator of the would-be-Goldstone bosons at zero momentum transfer. In particular, at the two-loop level, the photon would aquire a mass if the tadpole contribution would be omitted.

Our RG equations (6) for the v.e.v. v and the particle masses m are different from the ones obtained in the effective potential approach [9]. A comparison of predictions based on these two approaches have been recently performed in [10].

These structural considerations were important to check our calculations of the various counterterms.

For the calculation of the $O(\alpha\alpha_s)$ and the $O(\alpha_s^2)$ corrections to the top-quark propagator we refer to [4] and [11], respectively.

3. Numerical results

According to (1) we need to calculate propagator-type diagrams up to two loops on–shell. To keep control of gauge invariance we adopt the R_ξ gauge with three different gauge parameters ξ_W, ξ_Z and ξ_γ. For our calculation all diagrams have been generated with the help of **QGRAF** [12]. The C-program **DIANA** [13] then was used together with the set of Feynman rules extracted from the package **TLAMM** [14] to produce the FORM input which is suitable for the package **ONSHELL2** [15] and/or for another package based on Tarasov's recurrence relations [16]. The set of master-integrals, in the limit of massless lepton and light quarks (see details in [7]) includes diagrams with three different massive scales. In most cases exact analytic results in terms of known functions are not available. Thus, instead of working with the exact formulae (which only can be evaluated numerically, at present) we resort to some approxima-

tions, namely, we perform appropriate series expansions in (small) mass ratios. For diagrams with several different masses it is possible that several small parameters are available. In this case we apply different asymptotic expansions (see [17]) one after the other. Specifically, we expand in the gauge parameters about $\xi_i = 1$, in $\sin^2\theta_W$ and, for diagrams with Higgs or/and top-quark lines, in m_V^2/m_H^2 or/and m_V^2/m_t^2. Numerical results are obtained using the packages **ON-SHELL2** [15] and **TLAMM** [14]. Since the quality of the convergence of a series is not known a priori we have to calculate several coefficients of each expansion (six in $\sin^2\theta_W$ and five in mass ratios, $m_V^2/(m_H^2, m_t^2)$) to keep control on the convergence. For the one-loop diagrams and their derivatives we used the exact analytical results, as given in [18,3]. The expansion of diagrams with a top-quark and/or a Higgs boson leads to two-loop bubble diagrams with three massive lines (with two of the masses equal). For these master integrals we utilized a special form of representation [18,3]. The diagrams with massless fermion lines also demand special consideration. These diagrams develop threshold singularities which behave like powers of $\ln\sin^2\theta_W$. To control these terms we had to use the exact analytical results, which have been worked out in [3] using a technique developed in [3,18]. We found that after collecting the contributions from all diagrams the threshold singularities canceled. This is a manifestation of the infrared stability of the pole mass of the gauge bosons. The series expansion in $\sin^2\theta_W$ converges very well, and can be restricted to the first two coefficients. The expansion in the remaining mass ratios require three coefficients in order to get sufficient precision for light Higgs mass values. The numerics has been performed in MAPLE. To get control of the numerical stability, we run the MAPLE program with an accuracy of 100 decimals (a posteriori, as an experimental fact, we find that the minimal accuracy is 40 decimals).

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